

IMPROVED NUMERICAL METHODS FOR TURBULENT VISCOUS RECIRCULATING FLOWS*

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INTRODUCTION

The objective of the present study is to improve both the accuracy and computational efficiency of existing numerical techniques used to predict viscous recirculating flows in combustors. This paper presents a review of the status of the study along with some illustrative results.

The effort to improve the numerical techniques consists of the following three technical tasks:

- (1) selection of numerical techniques to be evaluated
- (2) two-dimensional evaluation of selected techniques
- (3) three-dimensional evaluation of technique(s) recommended in Task 2.

SELECTION OF NUMERICAL TECHNIQUES

Based on the criteria of accuracy, stability and boundedness the following discretization schemes were selected for evaluation in two-dimensional problems:

- (1) Second Order Upwind (SOU) differencing
- (2) Operator Compact Implicit (OCI) differencing
- (3) improved Skewed Upstream Differencing Schemes (SUDS).

To enhance computational efficiency the methods selected for two-dimensional evaluation include the Strongly Implicit Procedure (SIP), for solving for the pressure correction of SIMPLE or its variants, accelerated by the following techniques:

- (1) Conjugate Gradient (CG) acceleration
- (2) Block Correction (BC) acceleration
- (3) Additive Correction Multigrid (ACM) acceleration.

TWO-DIMENSIONAL EVALUATION OF SELECTED TECHNIQUES

Accuracy Improvement

Each of the selected techniques for improving accuracy was implemented adopting a conservative control volume approach and boundedness improvement strategies which ensured unique solutions and which posed no solution difficulties. The evaluation of the techniques for improving accuracy were carried out on a number of test problems including

- (1) transport of a scalar with a step profile at the inlet and uniform flow
- (2) transport of a scalar with a smeared step profile at the inlet and curved flow

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- (3) transport of a scalar with a unit source and uniform flow
- (4) transport of a scalar with a distributed source and uniform flow
- (5) laminar shear driven flow in a square cavity
- (6) laminar flow over a backward facing step
- (7) turbulent coannular flow.

For illustrative purposes results of problems 2 and 4 are presented in this paper. For problem 2, the predicted outlet profiles of the scalar, shown in Figure 1, are used to illustrate the degree to which each discretization scheme exhibits smearing of gradients and/or non-physical overshoots or undershoots (ie lack of boundedness). The results of problem 4, as shown in Figure 2 illustrate the accuracy and rate of convergence with grid refinement of each of the discretization schemes.

Implementing SOU in a conservative framework results in a scheme with negative influence coefficients which, in turn, introduce the possibility of overshoots and undershoots in the numerical solution, see Fig. 1. Some attempts, including the one described in reference 1, have been made to reduce or eliminate this unboundedness of SOU. However, all attempts to date result in schemes which either are not conservative or do not guarantee unique numerical results. Also, as shown in Fig 1., SOU results can exhibit smearing of gradients when relatively coarse grids are used. Figure 2 illustrates the second order rate of convergence of SOU on fine grids. However, the level of error of SOU is notably higher than that of other second order schemes evaluated in the present study.

At the outset of the present study there were several OCI schemes with a formal accuracy of fourth order available in the literature. However, these schemes were not conservative. The development of a conservative Control Volume based OCI (CV-OCI) was undertaken in this study. The result of this effort was a CV-OCI scheme of exponential type which exhibits a second order rate of convergence with grid refinement, see Fig. 2. Bounded CV-OCI solutions can be ensured for one-dimensional problems, but, as illustrated in Fig. 1, a CV-OCI scheme which ensures bounded one-dimensional solutions is not guaranteed to produce bounded solutions for multi-dimensional problems.

For problems where the Peclet number is high, both the SOU and CV-OCI schemes can be viewed as corrected Upwind Differencing Schemes (UDS). The corrections to UDS that are employed in these schemes are necessary to eliminate the numerical diffusion of UDS and its variants. However, for multi-dimensional flows, it can be shown that these corrections are responsible for the unbounded nature of the schemes. An alternative to the corrected UDS approach is to use SUDS. Although the numerical solutions of the original SUDS of Raithby exhibited no numerical diffusion, significant non-physical overshoots and undershoots were noted for some problems. also, the original SUDS was only a first order scheme. Recently, both Raw and Huget, see references 2 and 3, have proposed improvements to the original SUDS. These improvements include a Physical Advection Correction (PAC) to SUDS and, a flux element approach with two integration points per control volume face and a Linear Profile (LP) assumption along the flux element edges. The PAC is designed to improve the accuracy of SUDS by including the effects of diffusion and source terms on the advection term. The flux element approach was adopted to improve the boundedness of SUDS as well as to simplify the implementation of SUDS. The effectiveness of these improvements and other refinements developed throughout the course of this study are illustrated in Figures 1 and 2. Note the minimal overshoot and undershoot of LP-SUDS-PAC as well as its second order rate of convergence and low error levels.

In spite of the efforts to reduce overshoots and undershoots and to improve the accuracy of the discretization schemes, the SOU, CV-OCI and LP-

SUDS schemes are not sufficiently bounded to be used for predicting the turbulent kinetic energy and dissipation. It is essential that solutions for these turbulence quantities are bounded. Two schemes which ensure this boundedness are the bounded skew schemes BSUDS1 and BSUDS2, see reference 4. Unfortunately, since both schemes are bounded by including a sufficient component of UDS, accuracy improvements are noted only on relative fine grids. An alternative is to modify the LP-SUDS scheme to ensure that solutions are bounded. One such scheme is the Mass Weighted (MW) SUDS where the linear profile assumption along flux element edges is replaced by mass weighted averaging. Of course the boundedness of MW-SUDS, as illustrated in Fig. 1, is achieved at the expense of accuracy. Nevertheless, as can be seen in Figures 1 and 2, the accuracy of MW-SUDS is a significant improvement over the accuracy of UDS.

Computational Efficiency Improvements

The evaluation of the various techniques for enhancing computational efficiency were carried out on a number of problems including problems 5,6 and 7. Results for problems 5 and 6 are detailed in reference 5.

Preliminary evaluation of the convergence enhancement techniques revealed that conjugate gradient acceleration is very effective for coarse grids. However, it was found that the effectiveness of CG diminished significantly with grid refinement. This diminished effectiveness arises because substantial improvements in the numerical solution occur only after a sufficient and often excessive number of iterations have been performed such that all of the dominant orthogonal basis vectors of the solution have been set up.

The BC acceleration is, by design, effective when the solution for pressure correction is predominantly one-dimensional. Surprisingly, BC acceleration is also often effective even when a predominant one-dimensional component of the solution is not evident. For instance, the use of BC for problem 5 with a 48x48 grid results in a 40 percent reduction in CPU requirements.

The additive correction multigrid acceleration technique is designed to systematically account for components of the solution in all directions by ensuring that conservation is satisfied on coarse grids. The use of ACM acceleration results in reductions in CPU requirements similar to those of BC.

In addition to the evaluation of the techniques used to accelerate the procedure for solving for pressure correction, additional methods for improving efficiency were considered. One computationally expensive aspect of the existing solution procedures for incompressible flows is the method chosen to account for the coupling between pressure and velocity. The current SIMPLE-like methods such as the PISO variant of SIMPLER require an appropriate amount of underrelaxation and an excessive number of costly coefficient iterations. To overcome these difficulties the solution strategy which is employed by most SIMPLE-like methods was modified. Instead of performing the calculations for velocity and pressure correction, u^* , v^* and p' , only once before re-assembling coefficients, the SIMPLE-like method was repeated several times. Using this strategy, the parameter used to underrelax the momentum equations could be increased from 0.5 to 0.9 and the number of coefficient assembly iterations reduced by over 30 percent. Combining the new strategy with SIP-ACM reductions in CPU requirements of up to 70 percent were noted for laminar flows. For turbulent flows reductions in CPU requirements, ranging from 25 to 40 percent, were not as large.

RECOMMENDATION FOR THREE-DIMENSIONAL EVALUATION

Based on the results of two-dimensional evaluation outlined above it is recommended that the following techniques be implemented for three-dimensional evaluation

- (1) modified SUDS to improve accuracy
- (2) SIP-ACM to improve computational efficiency.

The implementation of modified SUDS will ensure that, on the coarse grids often used for three-dimensional predictions, significantly more accurate solutions than UDS with little or, if required, no overshoots and undershoots present. The ACM technique is recommended over BC because the former technique is expected to have a wider range of applicability and is expected to provide even more dramatic reductions in CPU requirements for three-dimensional problems.

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OUTLET PROFILES

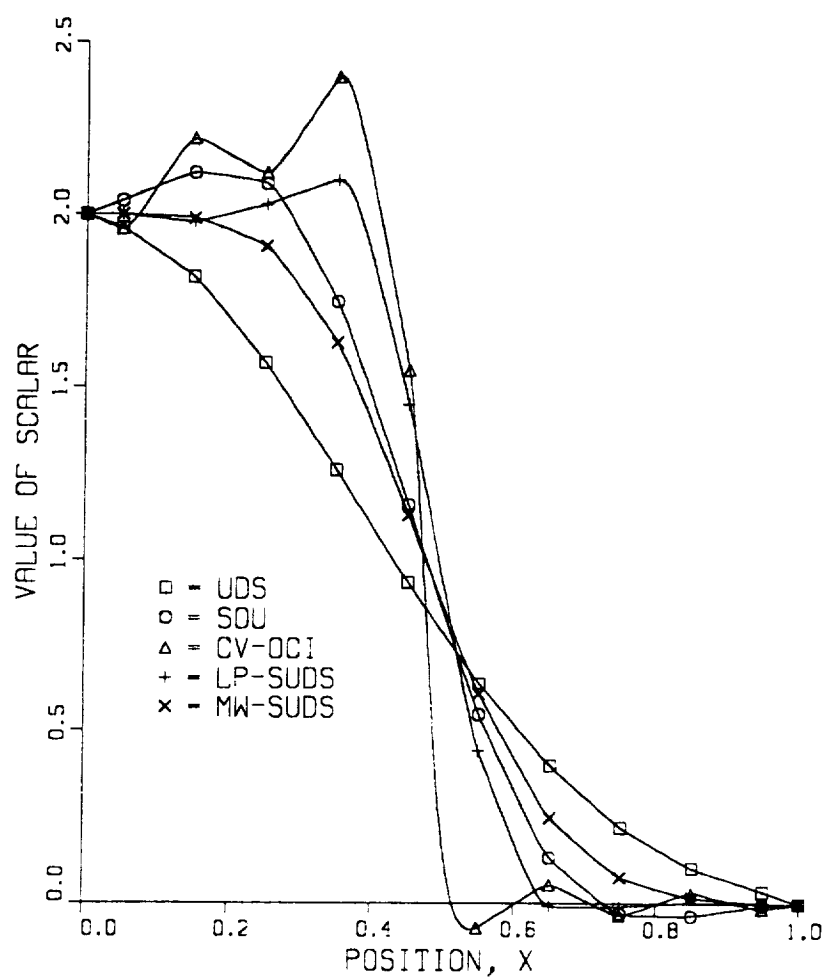


Figure 1: Outlet profiles for problem 2.

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RATE OF CONVERGENCE

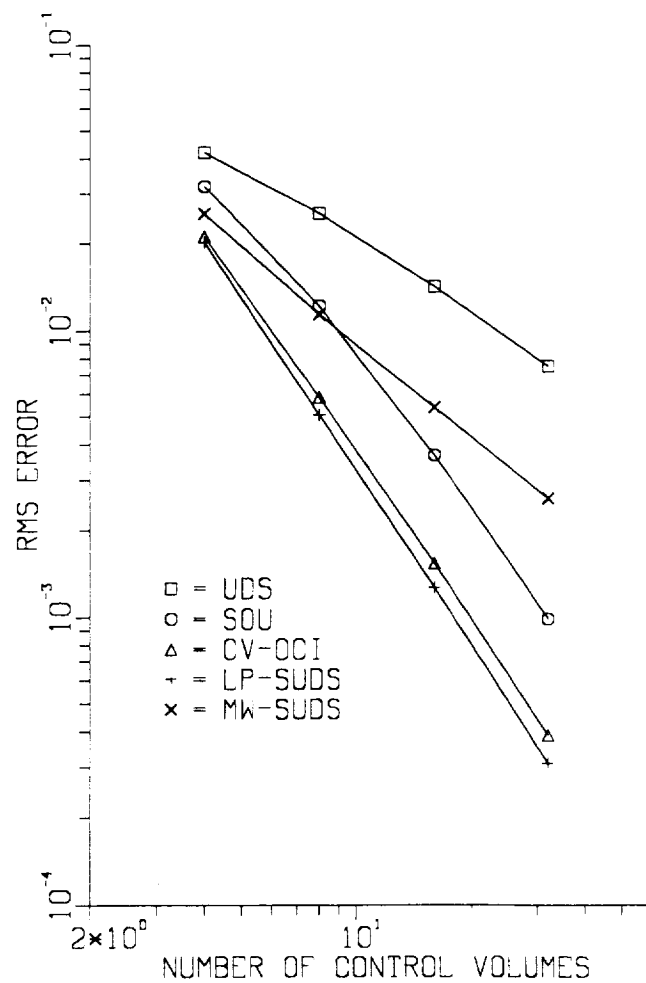


Figure 2: Convergence rate for problem 4.